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Abstract

Number talks are short mathematical discussions offering sensemaking opportunities for students. Aside from bolstering students' mathematical learning, this instructional routine may also support preservice teachers (PSTs) in investigating how to facilitate discussion-focused instruction. In this study, PSTs engage in a learning cycle to explore, plan and rehearse two separate number talks during human-in-the-loop simulations, and then reflect on these experiences. During the first simulation, PSTs focus on understanding the routine's components while positioning avatar-students as sensemakers as they elicit their participation. In the second simulation, PSTs build their instructional skills as they record representations of students' mathematical thinking, probe students' thinking in order to make mathematics visible, as well as notice missed opportunities to support students' mathematical reasoning during reflections of their experiences. Implications of this study suggest that simulations, when embedded within a cycle of enactment and reflection, support PSTs in developing professional noticing skills.

Introduction

Effective teachers facilitate discussions to build shared understanding of mathematical ideas (National Council of Teachers of Mathematics [NCTM], 2014) in ways that support students to talk with, respond to, and question one another as part of a math-talk community (Hufferd-Ackles et al., 2004, 2015). Further, current educational standards have embraced a vision of discussion-focused instruction (e.g., Council of Chief State School Officers, 2013). Yet, this vision is often different than what pre-service teachers (PSTs) have experienced as students (Lortie, 1975). In order to develop a vision of discussion-focused instruction, research (e.g., Goodson et al., 2019; McDonald et al., 2013) suggests that PSTs need practice-based learning experiences where they (a) explore and decompose how to lead a discussion, (b) approximate discussion by preparing, rehearsing, reflecting and refining an activity to enact with students, (c) enact the discussion-based activity with students, and (d) analyze the enactment. This practice-based approach to PST education may bridge the divide between learning *about* teaching and learning *to* teach (e.g., Ball, 2000).

Advocates for practice-based teacher education argue that PSTs should have ongoing opportunities to apply knowledge and note they are more likely to be effective when teaching is cultivated through opportunities to practice coupled with support and feedback (Benedict et al., 2016). Approximation of practice is a teacher education pedagogy that brings practice to life since it provides PSTs with opportunities to rehearse and reflect on teaching (Grossman, Hammerness, & McDonald, 2009). Further, Grossman, Compton, and colleagues (2009) argue that approximations are designed to control and reduce complexity of interactions so that PSTs have the opportunity to facilitate instruction in ways that support the development of their knowledge and skills for teaching. Yet, it is important to note that approximations of practice are deliberately different from working in classrooms with students and are not replacements or substitutes for these kinds of opportunities.

One way to support PSTs in applying their growing knowledge about teaching is through human-in-the-loop simulation (from here simulation). Simulations are a valuable tool that can support PSTs in learning how to elicit students' mathematical thinking (Lee et al., 2021), as well as facilitating discussion-focused instruction (Howell & Mikeska, 2021). During a simulation, PSTs are immersed in a virtual classroom that uses avatar-students controlled by a combination of artificial intelligence and a trained human actor. In this space, PSTs have a sense of immersion with both physical and social presence (Biocca et al., 2003), meaning that PSTs feel that they are in a classroom with students and that they have a meaningful impact on their learning (Hayes et al., 2013). Because of this immersion, simulations provide PSTs with the opportunity to rehearse what they are learning, reflect on it afterward, and rehearse the skill again (Dieker et al., 2014; Lee et al., 2021). Further, as PSTs interact with the avatar-students, they hone instructional practices while developing the content knowledge for teaching mathematics.

The current study investigates how simulations may be leveraged as a practice-based learning environment where PSTs approximate the work of teaching. In this space, PSTs rehearse number talks–short discussions around purposefully designed computation problems (Parrish, 2010/2014)–with avatar-students. This study aims to understand: What do PSTs notice during simulations that may support their developing teaching expertise?

Theoretical Framework: Noticing in Professional Practice

Noticing is about observing. In teaching, noticing is about intentionally seeing different aspects of professional practice (Mason, 2002) and supports the development of professional expertise (Jacobs et al., 2010). A growing body of research has explored how professional learning may support in-service teachers (e.g., Jacobs & Empson, 2016; Jilk, 2016; van Es & Sherin, 2008; van Es, 2011) and PSTs (e.g., Casey & Amidon, 2020; van Es et al., 2017) to notice details that are important to teaching and learning. In general, noticing is observed through the use of classroom observations, retrospective reflections, and examining responses to prompts in relation to artifacts (e.g., video and student work) because it is difficult to access noticing while it happens in the midst of instruction (Sherin et al., 2011). Further, noticing is about understanding how teacher negotiate salient features of teaching (e.g., Jacobs et al., 2010) and we have learned that noticing expertise can be improved with support (e.g., Sherin & van Es 2009; Star & Stickland, 2008). Yet, the literature notes a drawback when developing noticing skills with PSTs: Often PSTs tend to note the talk and actions of teachers more than notice students thinking when viewing classroom videos (Males, 2017). Therefore, PSTs may need different kinds of support to develop professional expertise that builds students' mathematical understanding.

There are several frameworks that describe teacher noticing (e.g., Jacobs et al., 2010; Mason, 2002; Sherin et al., 2008). These frameworks share two main components: (a) attending to details that are important to teaching and learning and (b) making sense of those details (Sherin et al., 2011). Yet, in teaching mathematics, noticing takes on an additional layer of importance because it influences the instructional decisions teachers make in responding to students. Therefore, this work draws upon the professional noticing definition stating that professional noticing occurs through the following processes: (a) attending to events in an instructional setting, (b) reasoning about these events, and (c) making informed teaching decisions based on the analysis of these observations (Jacobs et al., 2010; van Es & Sherin, 2008). Particularly salient to this study, is that the process of professional noticing is experiential, or practice-based in nature. Therefore, this process goes hand-in-hand with taking an intentional and inquiry-stance towards teaching that supports "picking up ideas", "trying them out", and "reflecting on" the outcomes (Mason, 2002, p. 30). Hence, this framework may be a lens to understand what PSTs notice about teaching and learning during simulated number talks.

Background

Number Talks: A Discussion Routine

Number talks are short mathematical discussions where students solve problems and share how they made sense of the problem (e.g., Parrish, 2010/2014). This discussion routine is designed to elicit students' mathematical thinking as teachers (a) set the stage by reminding students of the norms for number talks, (b) provide students time to make sense of the task, (c) gather student's individual responses, (d) facilitate a whole class discussion, and (e) summarize key ideas (Humphreys & Parker, 2015). During this routine, students enthusiastically respond by sharing their sensemaking strategies and the teacher encourages students to grapple, to comment, to question, and build on others' ideas. Because of this opportunity for sensemaking, the number talk discussion routine shifts the classroom community to a math-talk community (cf., Hufferd-Ackles et al., 2004, 2015) where students and their teachers investigate and talk about mathematics together (Woods, 2018).

Simulations as an Approximation of Practice

In order to prepare PSTs to enact number talks with students, they need opportunities to hone their skills for facilitating discussion-focused instruction, as well as learn to notice student's mathematical thinking. Further, we know that PSTs need different opportunities than observing to develop professional expertise. Therefore, an alternative to traditional rehearsals may be to incorporate simulations. Simulations combine real people and physical environments with virtual people and places that are controlled by humans (Murphy et al., 2018). During a simulation, PSTs interact with avatar-students in order to practice behaviors expected in their future careers (e.g., Hayes et al., 2013; Murphy et al., 2018). In this space, avatar-students respond to PSTs in real-time, allowing for meaningful interactions. Research is finding that this platform provides PSTs with the opportunity to learn instructional practices without placing *real* students at risk during the learning process (e.g., Dieker et al., 2014). Case studies support the idea that simulations may impact PSTs learning since they engage in targeted practice within authentic scenarios (e.g., Hayes et al., 2013). Further, quasi-experimental studies show that the skills targeted during simulations were transferred to real classroom settings, thereby positively impacting teacher practice (Straub et al., 2014; Straub et al., 2015).

Summary and Research Question

The present study builds upon the literature suggesting that PSTs need different supports to develop noticing skills than viewing classroom videos of discussion-focused instruction. Literature (e.g., Grossman, Compton, et al., 2009) argues that a practice-based approach where PSTs can approximate the work of teaching may develop these skills. Therefore, simulations may bridge the divide between learning *about* teaching and learning *to* teach without placing real students at risk during the learning process (Kaufman & Ireland, 2016). Further, embedding these approximations of practice during simulations provide a space for PSTs to experiment and grow, with an opportunity to receive feedback, debrief, and reflect on the enactments (Landon-Hays et al., 2020). To this end, this study investigates the following research question: What do PSTs notice during simulations that may support their developing teaching expertise?

Method

Research Setting and Data Sources

Eleven PSTs enrolled in a summer graduate course focused on learning to plan and deliver math lessons through a student-centered inquiry approach consented to participate in this instrumental case study (Creswell, 2013). The course was taught by the researcher and took place at a private southwestern university. For the three-credit graduate course, which was part of their requirement for state teacher certification, participants met face-to-face for three hours per day for 14 days spread across four weeks. All participants were women preparing to teach in the elementary (K-6) grades.

This study focused on their professional learning during *The Number Talk Project*. The learning objectives of the project were to (a) investigate and rehearse a discussion-focused instructional routine, (b) elicit and interpret individual students' mathematical thinking, and (c) orient students to each another and mathematics. To begin, the PSTs entered a learning cycle (e.g., McDonald et al., 2013) where they explored the number talk instructional routine and decomposed how to facilitate discussion-focused instruction through a unit of instruction. Then, they planned and rehearsed number talks during two separate simulations (at day 3 and 11) and immediately debriefed and reflected on their rehearsal (see Figure 1). The first number talk rehearsal consisted of PSTs enacting one of two quick images with the avatar-students, while the second number talk rehearsal consisted of PSTs enacting a number talk that they designed. During the time between rehearsals, PSTs investigated ways to engage their students in discussion-focused instruction by exploring talk moves–teacher actions that are effective in supporting students' mathematical thinking and learning (Chapin et al., 2009, 2013)–as well as understanding student's mathematical thinking across number properties and operations (Carpenter et al., 2015).

Three sources of data were collected and analyzed: (a) video and the corresponding transcripts of two rehearsals for each PST, (b) video and the corresponding transcripts of the whole group debriefs after each rehearsal, and

(c) individual written reflections (from here reflections). Each number talk rehearsal was video recorded and transcribed. The videos of the first simulations averaged approximately seven minutes in length for a total of 81 minutes and 29 seconds. The second set of simulations averaged approximately eleven minutes for a total of 125 minutes and 10 seconds. Whole group debriefs took place after the rehearsals and followed the format of discussing (a) what was learned about each student's mathematical thinking, (b) what was learned about eliciting student thinking, (c) what they learned about themselves as a teacher, and (d) goals for the next number talk based on feedback. The debrief after the first set of number talks was 20 minutes and 55 seconds, while the debrief after the second set of number talks was 20 minutes and 37 seconds. Individual written reflections mirrored the format of whole group debriefs, but first required that the PSTs watch the video of their rehearsal in order to promote reflective practice. There were two sets of reflections: one set after the first number talk and a second set after the second number talk.

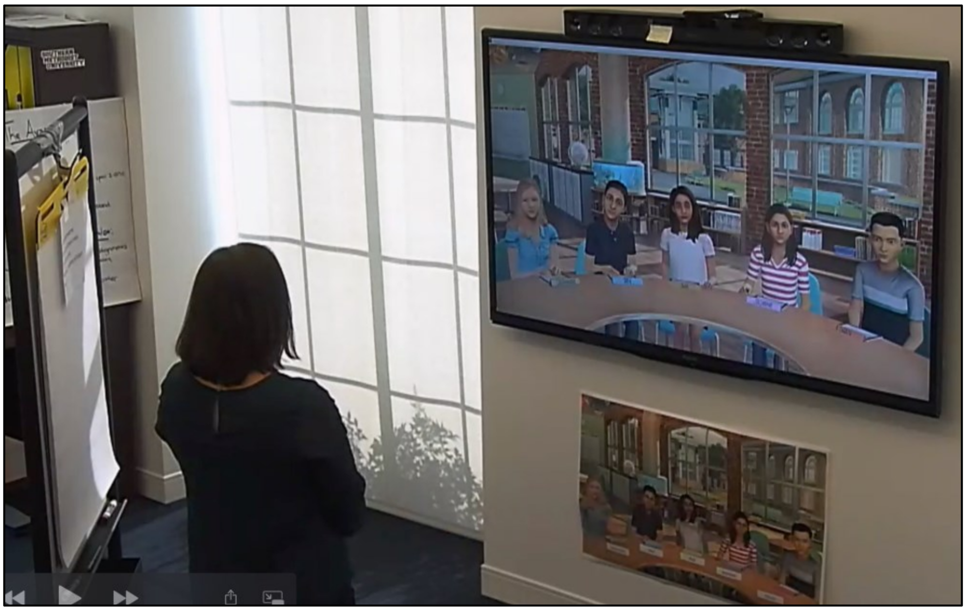

Figure 1. A PST Interacting with Avatar-Students During the Number Talk Project

Analysis

Data analysis consisted of three phases: (a) analytic memoing, (b) cycle of codes and coding, (c) and a creation of a time ordered matrix to explore how simulations supported PSTs to learn to enact discussion-focused instruction. All of the coding was conducted by the author and a trained graduate assistant, reconciling codes to reach intercoder agreement (Miles et al., 2014).

Phase one began by reviewing the corpus of data and creating analytic memos–a brief narrative documenting reflections and the thinking process about the data–while conceptualizing how the different forms of data tie together. Here, patterns of interaction, in particular the moves that the PSTs used to orchestrate a discussion, were noted in transcripts of each number talk simulation, debriefs, and reflections. These notes were used to define code choices and their operational definitions for the cycle of coding.

Phase two was a cycle of codes and coding. In this phase, the decomposition of leading a group discussion (TeachingWorks, 2021) was used as a framework to code the corpus of data since analytic memos highlighted moments when PSTs were either enacting these elements of discussion-focused instruction or were thinking about these elements during the debrief or in reflections. In the video/transcript data this meant that a discussion move followed by an avatar response was coded in terms of connections, correctness, eliciting, keeping discussion on track, orienting, and probing (see Figure 2). Each sentence of the debriefs and reflections were either coded into these categories or into one of three new codes added to include key takeaways (a.k.a. lessons learned), future goals, and other insights for leading a discussion-focused instruction (see Figure 3).

		Definition	Example
Framing	Launching	Moves the teacher makes at the beginning of a lesson to engage students in the following task.	Catherine: Today we're going to do a number talk. I am going to start by showing you a card with a certain amount of dots on it. I want everyone to think silently about how many dots they see and then I will ask you to raise your hand and let me know how you saw the dots. Everyone ready? Avatar-Students: [All nod their heads affirmatively.] Catherine: Great. Okay. Here is the first card. [PST shows dot card.] (Number Talk 1)
	Concluding	Moves teachers make to support students to remember or make sense of the content.	Amanda: Great thinking! Does everybody agree with Ava and Dev? That there's six. Avatar-Students: [All nod their heads affirmatively.] Amanda: Perfect. (Number Talk 2)
Orchestrating	Eliciting	Supporting students to participate by eliciting multiple ideas.	Rose: Ethan what do you see? Ethan: 01:19 Umm, [pause] I think it's five. (Number Talk 1)
	Orienting	Ensuring that students are attending, listening, and responding to the ideas of others.	Katie: Dev, can you um, repeat in your own words the way Ava solved it? Dev: She just swapped the one and the seven. (Number Talk 2)
	Probing	Probing students' thinking to clarify and expand student ideas.	Emma: How many boxes did you think about in your head? Jasmine: 12 boxes. Emma: Well ok. One, two, ... [drawing boxes on the whiteboard]. Jasmine: Yeah, like two rows of six...yeah like that. Emma: Okay. So, Andrew has seven and he needs how many more to get to 12. So how are you going to solve it with these boxes? Jasmine: You cross out the whole first row because its six. Emma: Okay. Jasmine: And then one more. So, one, two, three, four, five. Boom! (Number Talk 2)
	Connections	Supporting students to make mathematical connections.	Karlie: What do you all think? Do you think Ava's idea works with 20s? (Number Talk 2)
	Correctness	Assigning right or wrong to an idea.	Brooke: You're right about that! Good job Ethan. Ethan: Cool. (Number Talk 1)
	Keeping Discussions on Track	Moves to keep the discussion on track such as highlighting, redirecting, revoicing.	Emma: Eight. Okay, so let's count. Let's count them together. There's two, four, six and then there's one down here. So, six plus one is ... Ava: Oh, yea Dev was right. (Number Talk 1)

Figure 2. Codebook: Components of Discussion Focused Instruction

Note. Coding scheme adapted from the decomposition of the leading a group discussion (TeachingWorks, 2021)

Phase three began with the creation of a time ordered matrix–or intersection of lists–by time and code (Miles et al., 2014). The goal of this phase was to understand how the data converged by building a model to reveal what PSTs noticed during simulated number talks that may support their developing teaching expertise. Themes were assigned to describe the relationships across the three sources of data.

		Definition	Example from Data
Debrief and Reflections	Key Takeaways	Teacher moves and pedagogical concepts that teachers learned about as they participated in the project.	Chloe: I noticed that the wait time is really crucial. I think we get nervous if they don't respond in two seconds. MTE: I totally agree. Rose: It's like in that article too, like you get uncomfortable with silence. (Debrief 1)
	Goals	The goals teachers made to improve their practice between the different time points.	I wish I let Ava talk more the first time she spoke. She said she got four by seeing two sets of two. I responded that was similar to how Savanah saw it, two sets of two. I then started to move on, and Ava said two sets of eyes. I think it's really impressive that Ava made this connection to Savanah's strategy, and I wish I took the time to address that, or word my statement in a more open-ended way. Or just pose it as a question, asking if she thinks that it was similar to Savanah's, or similar or different to Savanah's? (Charlotte's Reflection 1)
	Other	Reflections on avatars and simulations	I think the concept of the avatars and their ability to respond and be so intuitive freaked me out too much at the time, and so I wasn't confident enough to address all of their responses. (Brooke's Reflection 1)

Figure 3. Codebook: Thematic Codes that Emerged from the Data

Findings

The following sections describe how PSTs focused on positioning the avatar-students as sensemakers as they began to notice their mathematical thinking during simulations.

Number Talk 1: Focus on the Number Talk Routine

During the first round of simulations, the math goal for the avatar-students was to perceptually subitize and flexibly decompose the dots in various ways while the PSTs focused on the basic structure of facilitating a number talk. As the PSTs presented the quick image to the five avatar-students (see Figure 4), they followed the number talk activity structure of (a) setting the stage, (b) making sense of the task, (c) gathering student responses, (d) facilitating a whole class discussion, and (e) summarizing key ideas and conclusions (Humphreys & Parker, 2015).

Figure 4. Quick Image

Note. This is the quick image that Tammy presented to the avatar-students. Other PSTs presented this image (or a similar image) or quick images of 10.

In what follows is an example of a typical *first* number talk within the simulated classroom. In this vignette, Tammy began by setting the stage.

1 Tammy: Good morning class! How are y'all today?

2 Ethan: Good, what are you holding?

3 Tammy: Some papers I'm about to show you. We're going to do a number talk today. What I'm going to do is show you a paper, and once I show it to you, I want you to think in your mind how many dots you see. Okay? Then I'll ask you to raise your hand after I've given you some time to think about it, and you'll tell me how many dots you think you saw. Does that sound like that makes sense? [Heads nod affirmatively.] Okay.

4 Tammy: Here's the first one. [Tammy holds up a dot card for a few seconds.]

5 Tammy: I'll show you one more time. [Tammy holds up a dot card again.]

In this scene, Tammy took the time to set the stage by launching the number talk in line 3. First, she explained to the avatar-students the goal of the number talk–"how many dots you see"–before beginning the activity. She also reviewed norms for discussion–"think in your mind" and "raise your hand"–before beginning the number talk. After, the avatar-students made sense of the task, she gathered student responses by eliciting the total number of dots that they saw on the image.

1 Tammy: So, who can tell me how many dots they saw?

2 Tammy: Ava?

3 Ava: Um, five.

4 Tammy: You saw five, did anyone else see a different number?

5 Tammy: Everyone saw five? [Heads nod affirmatively.]

Here, Tammy realized that all of the avatar-students readily subitized five dots on the card. She then decided to facilitate a whole class discussion by eliciting–encouraging multiple ideas–about *how* they saw the five dots.

1 Tammy: Okay, who can tell me how they saw five? [Dev raises his hand.]

2 Tammy: Dev?

3 Dev: I saw a box of four, then the one in the middle.

4 Tammy: Okay, so can you tell me a little more what you mean by you saw a box of four?

5 Dev: The outside makes a box.

6 Tammy: The outside...

7 Dev: Then there's four corners.

8 Tammy: So, you saw these four dots on the outside by the box, and then on the inside was one dot. Is that right?

9 Dev: Yes.

10 Tammy: Awesome!

In line 4, Tammy probes Dev's thinking in order to clarify and expand on his idea. She then, in line 8 uses the talk move of revoicing–the repeating, expanding on, or rephrasing a students' contribution to a conversation–to check if she understood him correctly, as well as to orient avatar-students to Dev's thinking as she begins to include other avatar-students in the conversation (Chapin, et al., 2013).

1 Tammy: Did anyone else see it differently?

2 Tammy: Jasmine?

3 Jasmine: I didn't count, it's just like when I roll the dice, it's a five.

4 Tammy: Oh, so you've seen this before on a dice [sic]? [Jasmine nods her head affirmatively.]

5 Tammy: Did anyone else see it that way? [Ethan raises his hand.]

6 Tammy: Ethan, you did?

7 Ethan: Yea.

8 Tammy: So, you've seen this on a dice before? [Ethan nods his head affirmatively.]

9 Tammy: So, who can tell me, in their own words, what that looks like on a dice? [Savannah raises her hand.]

10 Tammy: Savannah?

11 Savannah: What do you mean?

12 Tammy: So, tell me what that looks like to you, if you've seen that on a dice, can you tell me what that looks like to you?

13 Savannah: It's a diagonal line with extra dots.

14 Tammy: Oh, that's awesome! That's a completely different way of seeing that. So, did you see this diagonal line in the middle and then two dots on either side?

15 Savannah: Mm-Ehmm [Shaking her head affirmatively.]

16 Tammy: That's great. We all have different ways of seeing the number of dots on the page. Y'all did a great job! Thank you so much, I'll see you next time.

In this final scene, Tammy encouraged and supported other avatar-students to participate as she created space for a wide range of student ideas as she continued to elicit thinking. She worked on giving avatar-students time to speak while paying close attention to what the avatar-student was saying without unnecessary interruptions. Although Tammy’s summarizing key components and conclusion was brief in line 16, she took time to acknowledge that her avatar-students were sensemakers and had important ideas to contribute to the conversation. This verbal recognition positioned her avatar-students as capable.

In sum, this first experience with the avatar-students was a space where PSTs had the opportunity to facilitate a number talk in a low-stakes environment. In these data, all of the PSTs began the number talk by reminding the avatar-students of the norms as they set the stage, showed them the quick image, and provided them with time to think about the answer so that they could make sense of the task. Yet, there was variation in whether PSTs gathered student responses before eliciting thinking. Specifically, 5 out of the 11 PSTs immediately began eliciting individual avatar-students’ thinking before drawing out the total number of dots that they saw on the card. This was significant because accepting all initial answers, regardless of any errors, establishes a learning community (Parrish, 2010/2014). Further, since wrong answers are rooted in misunderstandings, ideas needed to be brought to the forefront so that thinking can be confronted and understood.

After this initial variation, all PSTs, as exemplified by Tammy, took up the challenge of encouraging and supporting avatar-students to participate. In these moments, each simulation looked and sounded like PSTs practicing how to elicit avatar-students’ thinking as they asked general questions intended to uncover mathematical ideas. At this time point, PSTs encouraged the avatar-students to share about the patterns that they saw in the quick image. When they probed deeper, the questions intended to draw out thinking, although the scope of questioning was limited because of the nature of the mathematical task.

Debriefing and Reflecting on the First Number Talk

The first debrief and written reflections centered around three main takeaways or themes revealed in the time-ordered matrix: (a) life-likeness of avatar-students, (b) positioning avatar-students as sensemakers, and (c) eliciting avatar-students thinking about a quick image. In order to give a sense of the what PSTs noticed and wondered about in these spaces, the following section highlights the features of PSTs understanding that supported their development.

Noticing Life-Likeness to Positioning Avatar-Students as Sensemakers

As the debrief began, the MTE asked the question, "What are you noticing and wondering about the simulations?" This question opened up the space for the PSTs to reflect on the avatar-students being so life-like. Amanda inquired, "How can they, like, I just don't understand how they can read the numbers and how they see what we were wearing", while Catherine noted, "the kids are a little bit different… they have distinct personalities" (Debrief 1). This theme carried over into the PSTs reflections. For example, Emma wrote in her reflection, "I thought this activity was very realistic and similar to what would happen in a real classroom" and Rose wrote, "It was crazy to see that the students were so receptive and had such attention to detail" (Reflection 1).

Once the novelty of simulation passed, PSTs began to notice that the avatar-students were sensemakers. During the debrief, the MTE asked, "Did you learn anything about these students today?" PSTs reported that, "they always came up with better ideas than what we had predicted (Katie)," and "they were [saying] things I never would have thought of (Chole)". Reflecting on her interactions with the avatar-students, Emma wrote:

> They are much smarter, more creative, and well behaved than I thought they would be. I did not give them the credit that they deserved. I re-watched the video of me doing my number talk, and I tried to pay close attention to what the students were saying and what they were picking up on. I think it is very cool how they come up with answers based on their past experience or memory from playing games…I thought this was really cool.

Emma's reflection was not unique. In fact, every PST described moments where they noticed that each student-avatar had a different way to think about the quick image presented during the number talk. PSTs also connected these different ways of knowing the number of dots represented on the quick image to the levels of thinking described in the learning trajectory for counting (Van de Walle et al., 2019). Further, PSTs noted that although avatar-students used different strategies, they ultimately arrived at correct answers. In sum, the debrief and the reflection revealed that PSTs noticed avatar-students' thinking.

Eliciting Thinking

As PSTs elicited avatar-students' thinking, they began to take action steps to better support them to make contributions to the whole group discussion. Here, two salient takeaways emerged from the data: PSTs noticed that they needed to (a) slow down (and reduce) teacher talk by making time for purposeful pauses, and to also

(b) prepare for discussions in advance so that they feel ready to support the avatar-students in-the-moment as they prompted sensemaking. During the debrief, several PSTs revealed that they needed to slow down the conversation by creating time for purposeful pauses. Catherine acknowledged, “I think I learned that I need to slow down … I mean it’s about wait time, and practicing to slow down and just wait for everyone to engage (Debrief 1)”. Adding on, Rose offered, “I began to count in my head to make sure I give them [enough] wait time (Debrief 1)”. This theme is also found in the reflections: 7 out of the 11 PSTs mentioned that as they watched videos of their first number talk, they wished they had paused longer in order to give avatar-students time to think before they moved to the next question or another student. Further, they stated that practicing these purposeful pauses was a goal that they planned to implement during the next simulation.

As PSTs discussed the importance of providing avatar-students with enough time to fully explain their thinking without interruption, they wondered about the balance of being prepared with the *right* questions to interject when the conversation slowed or mathematical connections needed to be made. In the reflections, 9 out of the 11 PSTs discussed the importance of preparing open-ended questions for number talks. Salient to this point is Katie’s reflection:

> For the next number talk, I hope that I ask more open-ended questions. While I was in the moment, I didn’t realize that I was often asking yes or no questions, and sometimes putting words in students’ mouths. I need to ask them to explain themselves, rather than trying to do it for them. I think that I was honestly just so relieved whenever they said something I was expecting and had planned for, I got overly excited and tried to prompt them to say something else.

In this moment, Katie realized that she took control of the conversation and decided instead to ask open-ended questions in order to support her avatar-students’ sensemaking. In a similar moment, Yasmine realized that when she engaged in a conversation with avatar-students that she was eager to take ownership of the discussion and concluded, “I should allow them more time to finish their thoughts before I jump in and explain to them.” She also wrote, “I think the purpose is for students to engage in discussion, exchanges, thoughts, and mistakes. I will try to be more interactive and ask open-ended content questions so the student can share.” Relatedly, Tammy’s reflection revealed insight into her simulation. She stated, “I still need to practice encouraging the students to think more deeply about the problem and creating opportunities to foster their engagement with one another.” In sum, PSTs realized that they need to prepare number talks in a way where they could employ different talk moves to orient students to one another and the mathematics of the number talk while supporting avatar-students to reason about and add on to their peers thinking. Yet, they noticed they needed to relinquish control of the conversation in order to support sensemaking through open-ended questions and peer-to-peer engagement in discussion.

Number Talk 2: Focus on Making Mathematics Visible

Much like the first round of number talks, PSTs continued to focus on the basic structure as they enacted a number talk that they planned around a specific mathematical standard. These number talks ranged from contextual word problems, to quick images, to naked problems. In the vignette that follows, Tammy designed a contextual word problem: “Lila picked 8 strawberries. Cooper picked 4 times as many strawberries as Lila. How many strawberries did Cooper pick?" This problem connected to the third grade Common Core State Standard 3.OA.5 which applied the properties of operations as strategies to multiply and divide (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The focus of the analysis will be in response to the instructional goals that Tammy made in her reflection. That is, Tammy, besides representing student thinking on the whiteboard, intended to incorporate additional talk moves such as asking for evidence or reasoning, repeating what another student had said, or asking if they agree with what another student had said as she pressed avatar-students to make connections between the properties of multiplication. After Tammy welcomed the avatar-students, set the stage, and provided plenty of time for avatar-students to make sense of the task, she gathered avatar-students responses. Answers ranged from 32 to 34. She began the discussion by asking Jasmine to defend her answer. This is where the following scene begins.

1 Jasmine: Um, well I, I thought that it would be easier to do eight times two first.

2 Tammy: Eight times two, okay. [Writing Jasmine's solution on the whiteboard. Tammy continues to represent each student's strategy on the whiteboard.]

3 Jasmine: And then that's 16.

4 Tammy: Okay.

5 Jasmine: And then I did eight times two again.

6 Tammy: Eight times two again. Okay. And that's sixteen.

7 Jasmine: And then I added them.

8 Tammy: And then you added them, okay. And how did you add them together?

9 Jasmine: Um, I dunno … I just started with 15. And I just did 15 plus 15 because I know that's 30.

10 Tammy: Okay. 15 plus 15 is 30, okay, and then what?

11 Jasmine: Uh, that's it.

12 Tammy: Okay, and I think you said you got 34, right?

13 Jasmine: Oh, right. I think I may have added wrong…

14 Tammy: Okay, well let's see how Dev solved [the problem] and maybe we can see the difference [between the two solutions]. Okay, Dev, tell us what you did.

15 Dev: I actually broke it down like Jasmine, so I think I can help.

16 Tammy: Okay, awesome. Tell her what you did.

17 Dev: Okay, so, Jasmine, I did the eight times two, twice, like you did. So, eight times two is 16, right?

18 Jasmine: Yeah.

19 Dev: And then you do eight times two again, so it's 16 again.

20 Jasmine: Yeah.

21 Dev: And then you, you just add the 16 plus the 16.

22 Jasmine: Right, I did it with the 15…

23 Dev: Right, yeah because you know that number, because you know with the fives it's an easy number. But you know six plus six is 12.

24 Jasmine: Yeah.

25 Dev: Yeah, so then you just, you know that number, so then you just add the other tens. Do you know what I mean?

26 Jasmine: I think, maybe.

In this scene, Tammy supported both Jasmine and Dev's reasoning by giving them the space to explain their own thinking with the goal that they provide evidence that shows that they were reasoning about the answer. Both Jasmine and Dev worked to explain how they decomposed four into two groups of two in order to arrive at the answer. However, since Jasmine was still confused about where she may have made a mistake, Tammy took the opportunity to bring Ava into the conversation by having her repeat what Dev was saying in her own words.

1 Tammy: Ava, can you explain what Dev was just saying in your own words?

2 Ava: Yeah, like he was trying to break it down so you're working with smaller numbers. So instead of just working with two sets of 16 cause that's super overwhelming, then you just do six and six.

3 Tammy: Six and six, okay.

4 Ava: And that's 12.

5 Tammy: That's 12, and where did those sixes come from Ava?

6 Ava: Those are the single numbers in the one's place.

7 Tammy: Like from here? 16 and 16. Alright, and then what?

8 Ava: Well, but then you've got a whole bunch leftover [numbers]. You know you still have 20 because you have the tens places.

9 Tammy: So, are you saying that we could think about 16 like ten plus six?

10 Ava: Totally, yeah that's absolutely right.

11 Tammy: Okay, so we could have ten plus six, plus ten plus six, is that right?

12 Ava: Yeah.

13 Tammy: Okay. Okay, so Jasmine could think about it like the way Dev was saying?

14 Jasmine: Yeah, yeah! I just didn't get it at first, but when you wrote it down, I can see it!

15 Tammy: So, ten plus ten is how much?

16 Jasmine: 20

17 Tammy: And then six plus six is?

18 Jasmine: 12

19 Tammy: 12, and that gives us …

20 Jasmine: 32!

21 Tammy: Alright

22 Jasmine: [Giggles.]

In this scene, after Ava explained Dev's thinking in her own words, Tammy checked in with Jasmine to make sure she was making mathematical connections by supporting her to compare her thinking to what Dev was explaining to her. Most importantly, Tammy in line 13 provided the space for Jasmine to revise her thinking. In order to move the discussion forward, as well as add more voices to the discussion, Tammy elicits Savannah and Ethan's thinking.

1 Tammy: Savannah or Ethan, did y'all solve this differently?

2 Savannah: Mm-Ehmm [Shaking her head affirmatively.]

3 Tammy: Savannah how did you solve the problem?

4 Savannah: I didn't break it into chunks like that. I just did eight times four.

5 Tammy: Eight times four. Savannah thought about a math sentence that could go along with this problem. She thought eight times four equals, what?

6 Savannah: 32

7 Tammy: 32. So you knew a [multiplication] fact, is that what you're saying Savannah?

8 Savannah: Yeah!

9 Tammy: Okay, good, what about you, Ethan?

10 Ethan: I did it the super long way.

11 Tammy: Okay, I like long ways, tell me the long way.

12 Ethan: Yeah, so I was just like counting in sets of eight.

13 Tammy: Sets of eight, okay so what did that sound like to you?

14 Ethan: Well, you know, like, (laugh) it was super long, like 9, 10, 11, 12, 13, 14, 15, 16, you know like really just counting out.

15 Tammy: So, you started with nine, and then you counted individually one by one …

16 Ethan: Yeah, like I knew I had to do it four times, so every time I got to like a chunk of eight, that was another one.

17 Tammy: Awesome, so you thought eight plus eight plus eight plus eight?

18 Ethan: Yeah!

19 Tammy: Okay, good, I have a question. So, Savannah and Ethan brought us to an important point. Savannah said that eight times four is 32, but Ethan is saying that four eights are 32. So, is this the same if I write it this way? [Tammy writes four times eight equals 32 on the whiteboard].

20 Ethan: No way!

21 Tammy: Do y'all agree with Ethan? Is this the same as eight times four, or different? [Pause.] Would it still give you 32? [Pause.] Dev?

22 Dev: Yeah, it's the same thing with multiplication, you can change the order, it doesn't matter.

23 Tammy: Why do you think that? Tell us how you know.

24 Dev: Um, cause you're not subtracting them, like it's just, it's a different formula. You get the same number no matter how you mix 'em up.

25 Tammy: Okay, Jasmine could you add onto that a little more?

26 Jasmine: Um, yeah, I think Dev's trying to say that you can, you can put the numbers in any order.

27 Tammy: In any order, okay.

28 Jasmine: Cause the total, it equals 32 either way.

29 Tammy: So, are you saying is that if we have eight sets of four, you're saying that is the same as four sets of eight?

30 Jasmine: Yeah, yeah!

31 Tammy: What do you think about that Ethan?

32 Ethan: Uh, you are blowing my mind.

33 Tammy: Okay, well we're going to continue this conversation tomorrow. You all did a really good job …

In this final scene, Tammy took the opportunity to make mathematics visible and supported her avatar-students to make mathematical connections by discussing the commutative property of multiplication as she made connections between Savannah's and Ethan's thinking. Although, Tammy did not name the property in line 19, she facilitated a conversation about the underlying mathematical properties that the avatar-students were using to make sense of the solution strategies. Further, she worked on her goal of supporting avatar-students' ability to reason with each other by asking if they agree or disagree and also by adding on to the conversation.

In sum, the second set of simulations provided a space for the PSTs to build upon the skills that they gleaned from the first set of simulations, debrief, and reflections. Like Tammy, the other PSTs were already familiar with the number talk routine, valued avatar-students as sense-makers, and had practiced how to elicit student thinking. Therefore, the second round of number talks provided a practice-based space where they could strengthen and build upon these skills as they worked to make the mathematics embedded within the number talk visible for all avatar-students.

Debriefing and Reflecting on the Second Number Talk

The second round of debrief and reflections, built upon the instructional practices that PSTs gleaned from the first simulations: positioning avatar-students as sensemakers and eliciting avatar-students' thinking. Additionally, they began to develop the following instructional practices: (a) recording representations, (b) making mathematics visible, and (c) noticing missed opportunities. Figure 5 is a model depicting how these teaching practices developed and overlapped during the two rounds of simulations. The following sections highlight how PSTs made sense of these additional instructional practices.

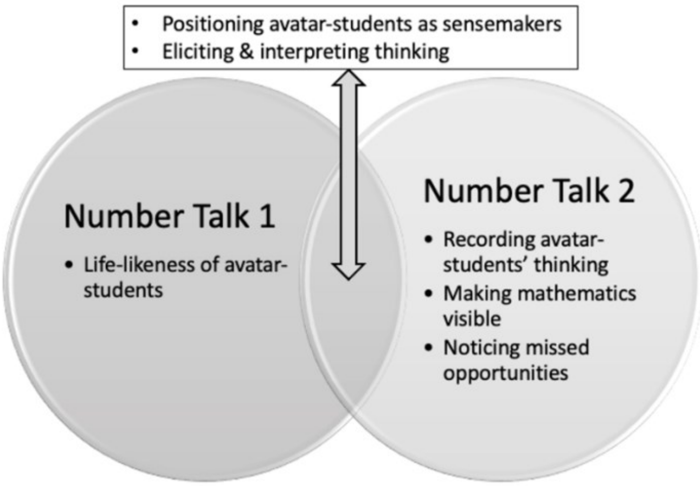

Figure 5. HIL Simulated Number Talks Supported PSTs In Developing Teaching Expertise

Recording Representations

As PSTs reflected on their second number talk, they noticed the value of recording and representing student thinking. In these discussions, PSTs realized the importance of visual representations to help students make sense of problems. Salient in these data was how the act of recording avatar-student's responses not only supported students in making mathematical connections, but also prompted PSTs to make sense of their thinking. For example, during the debrief Catherine expressed, "We couldn't really do it here, but with actual kids you could say, 'Can you come up and show us how you got this?' And they could point out their thinking

(Debrief 2).” In this moment, she brought forward the value in supporting students to express their thinking across multiple modalities as they participate in meaningful discourse. In addition, this may signal that this teacher move may give her the opportunity to understand her students’ contributions to the discussion. In fact, 6 out of 11 PSTs reflections examined the pedagogy around recording student thinking. Representative of these data was Amanda’s reflection, “I really liked being able to write down their thinking on the board. It not only helped them explain their ideas to me, but it helped me keep track of what each student was saying and allowed me to come back to someone if I needed to (Reflection 2).” In sum, PSTs were beginning to recognize the important role of representations in the sensemaking process and how they could use this public record to inform how they orchestrated the discussion.

Making Mathematics Visible

As PSTs participated in simulations, they began to realize the complexity of the discussion-focused instruction in terms of responding to avatar-students’ thinking that was made visible through the talk moves that they employed. A few of the PSTs worried that they were not ready to take on the role of a facilitator in a student-centered classroom. For example, Chloe stated, “I was more concerned they were going to throw something at me and I wouldn't be able to figure out. And that, that was, that was kind of the uneasy part (Debrief 2)”. Catherine expressed, “I had no idea what he was saying” as she was talking about her rehearsal. This idea was echoed in 3 of the 11 reflections. For example, Rose stated, “I will try not to be intimidated of receiving leftfielder questions from the students and remember that we can work out the problem and learn the method together (Reflection 2)”. Here, Rose recognized her fear, but realized that because of the skills she had developed, such as talk moves and pedagogical content knowledge, she’d be able to make sense of her students’ mathematical thinking.

Realizing the skills that they were developing, the PSTs reflected on the teacher moves designed to probe, clarify, and expand on students’ mathematical ideas. In fact, all 11 PSTs reflected on ways to press on avatar-students’ thinking in order to make mathematics visible. A representative summary of these moments was found in Yasmine’s reflection. She wrote, “By defending their responses or even expanding on existing ideas, it not only means that the student truly understands the theory/concept behind it, but also it required more higher-order thinking skills (Reflection 2)”. However, PSTs also realized that they still had room to grow in developing their instructional skills. For example, Tammy wrote:

> I think I did a good job of asking deeper questions this time, but [I] want to refine my ability to repeat and progress students’ thinking by reinforcing mathematical concepts along the way. For example, when Ava mentioned the “single numbers” [that] she added, I could have referenced the ones and tens places as I drew her thinking on the board (Reflection 2).

Noticing Missed Opportunities

Connected to Tammy’s reflection, was the opportunity for PSTs to notice missed opportunities. Missed opportunities (van den Kieboom et al., 2017) are moments when a PST should have explored their students’ thinking but failed to do so. In fact, after the first number talk rehearsal, only 1 out of 11 PSTs reflected upon a missed opportunity that she attempted to support, but was not sure how respond in the moment. During the second round of number talk rehearsals, 7 out of 11 PSTs described moments where they should have seized opportunities to make mathematics visible in their reflections. Representative of these data is Charlotte's reflection on her number talk. She wrote, "I wish I made a connection to the commutative property [of addition]. I should have asked the students how Jasmine's strategy and Dev's strategy are similar and/or different (Reflection 2)." She continued:

> I learned a lot about myself as a teacher in this one moment. Since I prepared this lesson and also prepared questions, I went into this number talk wanting the students to realize what Jasmine understood. However, I ended up being too stuck in this mindset and didn't even realize how much learning the students would have gotten out of the connection between Dev and Jasmine's strategies.

This reflective moment highlights a shift in PSTs instructional practice. That is, they are beginning to realize that merely engaging students in a discussion is not enough: they need to support students to meet the math goal of the lesson by seizing opportunities to support sense-making that ultimately builds conceptual understanding. In sum, PSTs were beginning to notice that as they probed the avatar-students' thinking that they supported their developing mathematical skills. Further, they noticed that there was room to continually improve in how they made the mathematics visible to their students.

Discussion

Practice-based teacher education literature revealed that before PSTs lead discussion-focused instruction with students, they need opportunities to hone their skills during experiences that approximate the work of teaching (Grossman, Compton et al., 2009). From the literature, we know that PSTs need opportunities to intentionally reflect and learn from experiences (Mason, 2002) and that noticing expertise can be improved with support (e.g., Sherin & van Es 2009; Star & Stickland, 2008). Further, PSTs should have opportunities to develop these skills early in their teacher preparation program (van Es, 2011; van Es & Sherin, 2002).

Yet, PSTs need different supports to shift from noticing the talk and actions of teachers (Males, 2017) to noticing students' thinking about specific mathematical content (van den Kieboom et al., 2017). New research on approximating the work of teaching within simulations has found support for PSTs in learning how to elicit students' mathematical thinking (Lee et al., 2021) and to enact discussion-focused instruction (Howell & Mikeska, 2021). Building on these studies, the findings of the present study suggested that simulated number talks, coupled with reflective practices, supported PSTs in noticing details that were important to teaching, as well as details about students' sensemaking, in a way that builds expertise for teaching.

Supporting Noticing and the Development of Expertise for Teaching

The first set of simulations familiarized PSTs with avatar-students and to the number talk discussion routine. Because of the life-likeness of the avatar-students, the PSTs in the present study, as well as in previous research (e.g., Hayes et al., 2013), felt they were in a real classroom with students, and that they impacted their learning.

This opportunity for PSTs to teach in the virtual classroom extends the noticing literature beyond using video (van Es et al., 2017) or role-plays (e.g., Males, 2017) since role-plays may not represent an authentic approximation of a classroom environment (Storey & Cox, 2015).

In these data, there were several noteworthy advantages to simulations that may have supported PSTs in developing teaching expertise. When number talks were rehearsed with the avatar-students, PSTs had an authentic opportunity to practice discussion-focused instruction without placing real students at risk during the learning process (Dieker et al., 2014). Since PSTs observed each other and had the opportunity to debrief together, this shared experience provided a space where key features of the number talk routine were investigated. For example, during simulations for number talk 1, 5 out of 11 PSTs immediately began eliciting students-avatars' thinking before drawing out the total number of dots that they saw on the card. During the debrief, PSTs examined the instructional activity and considered why it was important to accept all initial answers. Specifically, they considered that this feature of the number talk routine supported the establishment of a learning community, while bringing misunderstandings to the forefront so that they could be confronted and understood (Parrish, 2010/2014). Further, during the debrief and reflections PSTs also examined the kinds of questions that worked best to elicit and probe students' mathematical ideas.

This platform also provided PSTs the ability to interact with the avatar-students in a way that recognized the relational work of teaching since there was presence and suspension of disbelief as PSTs temporarily accepted the avatar-students as a real class of students (Hayes et al., 2013). Salient to this experience were the moments when PSTs shared their uncertainty about taking on the role of a mathematics teacher since they were concerned about not being able to respond to students' mathematical thinking. Because simulations offered up life-like experiences, PSTs described moments where they noticed that each avatar-student had a different way to think about the mathematics discussed during each of the number talks. This positioned the avatar-students as sensemakers and was an important finding since traditional rehearsals, where peers role-play students, may not have provided the same opportunity for PSTs to develop noticing skills in relation to students' mathematical thinking (Males, 2017; Storey & Cox, 2015). Further, this opportunity to notice students' mathematical thinking, supported PSTs to connect different ways of knowing to the mathematics learning trajectories (e.g., Van de Walle et al., 2019) that they were learning about during methods instruction. In sum, simulations provided PSTs the opportunity to learn to notice since they (a) attended to events within the virtual instructional setting, (b) reasoned about these events, and (c) made informed teaching decisions based on the analysis of these observations (Jacobs et al., 2010).

The second number talk, debrief, and set of reflections built on what PSTs were noticing by describing their growing attention to the instructional practices and skills that they needed to develop in order to facilitate discussion-focused instruction. In these data, PSTs noticed that recording representations of students' thinking, probing students to make mathematics visible, as well as noticing missed opportunities to extend students' mathematical learning were important skills needed to successfully conduct a number talk discussion. One instructional practice PSTs noticed was of particular importance was the role of a public record or written mathematical representation of the avatar-students' thinking. In these data, 6 out of 11 PSTs noticed the important role that public record played in building a math-talk community (Hufferd-Ackles et al., 2015), supporting sense-making through a variety of solutions and representations of these strategies (Mainali, 2021), and also how they could leverage it to inform the path of discussion-focused instruction (Arcavi, 2003; Fuson & Murata, 2007; Stylianou & Silver, 2004).

Also, during the second round of number talks, PSTs realized that number talks were not lectures or speeches. Particularly salient was that all 11 PSTs noticed and reflected on ways to press on avatar-students' thinking in order to make mathematics visible. In these data, PSTs noticed the importance of supporting avatar-students to expand and revise their thinking (Jansen et al., 2016), their role in posing purposeful questions that supported them to expand on the mathematics goal of the lesson (NCTM, 2014), and what they would do differently the next time they facilitated discussion-focused instruction. As PSTs watched their simulations and reflected on their enactment of their number talk, they also realized missed opportunities. Recall that missed opportunities were moments when a PST should have explored their students' thinking but failed to do so (van den Kieboom et al., 2017). Salient was that during the first round of number talk reflections only 1 out of 11 PSTs reflected upon a missed opportunity to make a mathematical connection because she was not sure how to respond in the moment. Yet, during the second round of number talk rehearsals, 7 out of 11 PSTs began to notice opportunities where they could make connections to mathematical properties or support students to explore a mathematical idea in a deeper and more meaningful way.

The Role of Debriefs and Reflections

A key advantage to developing the skills needed to facilitate discussion-focused instruction was the opportunity that PSTs had to immediately debrief after their simulations with their peers and instructor (Grossman, Compton et al., 2009). Aside from the shared experience, PSTs had opportunities for feedback–reactions from the avatars, feedback from the teacher educator and fellow students–that can support them in noticing salient characteristics of mathematics instruction (e.g., Aparicio et al., 2021). Further, the opportunity to watch videos of their simulations may have supported PSTs to notice what was occurring during the number talk (Coffey, 2014; Sherin & Van Es, 2005). Taken together, another advantage of the simulations was that the practice-based nature of this approximation of practice supported PSTs in taking an inquiry-stances towards their practice (Mason, 2002) and thereby connecting theory about teaching and learning to the practice of teaching,

Implications for Practice

As teacher preparation programs shift to include opportunities for PSTs to learn from practice, this study provides evidence that simulations have the potential to provide an authentic approximation of practice before stepping into a classroom and providing live instruction (Murphy et al., 2018). Specifically, the results of this study provide evidence that simulations afford the opportunity for PSTs to rehearse discussion-focused instruction in ways that supported them to notice about self and students. Further, this study calls attention to the importance of developing ways to foster reflective practice that foregrounds noticing.

Limitations and Suggestions

While this study provides a case for the promise of using simulations to support the practice-based work of teacher education programs, the study was limited to a small number of elementary teachers at a private university. Therefore, this work may not be generalizable to other teacher preparation settings, especially if class size is large. Also, there may be a limitation to the targeted practice that is afforded by the simulations because they may not prepare PSTs for misbehavior, unexpected questions, and interruptions that may occur with large groups of students.

Although this study offers insight into the what PSTs notice during simulated number talks, future research is needed in order to examine (a) the anxieties that PSTs bring to teaching mathematics and how we can best support them to overcome their apprehensions, (b) the number of cycles of investigation, enactment and reflection needed to support PSTs in developing noticing skills needed for developing expertise for teaching mathematics, and (c) if PSTs take what they learned in simulations and apply it to their (future) work with students in real classroom settings.

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